The 37th Science Olympiad for Young Mathematicians, Physicists and Chemists

January 2025
Rudbeckianska upper secondary school
Västerås, Sweden

Physics - Form 11

Question 1. Passive solar house **(10 points)**

A house uses solar energy to heat 50 glass containers to 80 °C during the day. Each glass container is filled with 20 L of water. The house loses heat at an average rate of $50\,000\,\mathrm{kJ\,h^{-1}}$ during a 10 h long winter night. A thermostat-controlled 15 kW back-up electric heater turns on whenever necessary to keep the house at a constant temperature of 22 °C.

(a) How long did the electric heater run at night? (7)

(b) How long would the electric heater need to run if the house did not have solar heating? (3)

Hint: The specific heat capacity of water is $c_v = 4.18\,\mathrm{kJ\,kg^{-1}}$.

Solution:

(a) Assume that the house and water containers are a closed system. Total energy lost to the environment during the 10 h long night is

$$E_{\text{out}} = 5 \times 10^5\,\mathrm{kJ} \qquad \text{(1 point)}$$

Energy added from the heater is

$$E_{\text{in}} = P_e \Delta t \qquad \text{(1 point)}$$

where $P_e = 15\,\mathrm{kW}$. The change of the internal energy of the system (1 point)

$$\Delta U = E_{\text{in}} - E_{\text{out}}$$

The change of internal energy of the system comes from the change of internal energy of the water containers (since the energy of the air and the rest of the house is assumed to be unchanged as a consequence of constant temperature). The total mass of the water is $1 \times 10^3\,\mathrm{kg}$ (1 point) hence

$$\Delta U = mc\Delta T \qquad \text{(1 point)}$$

and

$$\Delta t = \frac{mc\Delta T + E_{\text{out}}}{P_e} = \frac{1000 \cdot 4.18 \cdot (22-80) + 500000}{15} = 17\,171\,\mathrm{s} = 4.77\,\mathrm{h}$$

(1 point for correct expression and 1 point for correct value in hours).

(b) If the house did not have any solar heating then $\Delta U = 0$ (1 point) and hence

$$\Delta t = \frac{E_{\text{out}}}{P_e} = \frac{500000}{15} = 33\,333\,\mathrm{s} = 9.26\,\mathrm{h}$$

(1 point for correct expression and 1 point for correct answer in hours).

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Question 2. Force between boxes **(10 points)**

Two boxes are placed on a inclined plane. The mass of box A is $m_A = 25\,\mathrm{kg}$ and box B has mass $m_B = 15\,\mathrm{kg}$. The friction coefficient between the surface of the inclined plane and the boxes is for box A $\mu_A = 0.7$, and for box B $\mu_B = 0.3$. What is the force between the two boxes right after they are released? **Hint:** Assume that they move together down the slope and set $g = 9.82\,\mathrm{m\,s^{-2}}$.

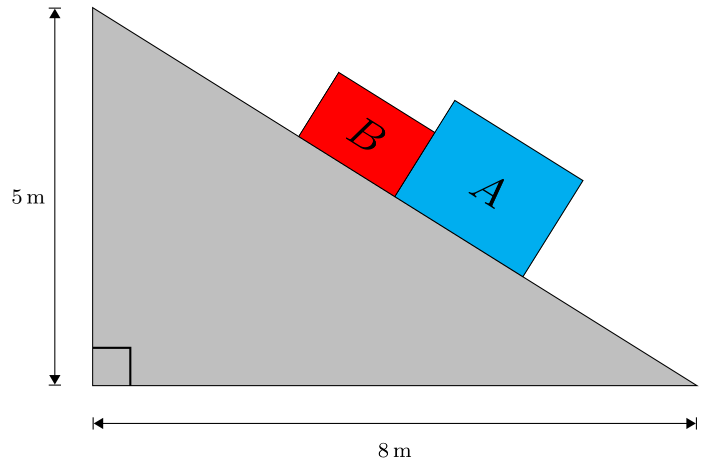


Figure 1: The problem setup and the geometry for the two inclined plane and the two boxes A and B.

Solution: Define the coordinate system with the x-axis parallel to the inclined plan and the y-axis as the normal to the inclined plane.

We introduce the angle θ as the angle between the inclined plane and the "ground". The force on box B is given by:

$$\sum F_y = N_B - m_b g\cos\theta = 0 \implies N_B = m_B g\cos\theta$$
$$\sum F_x = -\mu_B N_B - F + m_B g\sin\theta = m_B a$$

The force on box A is given by:

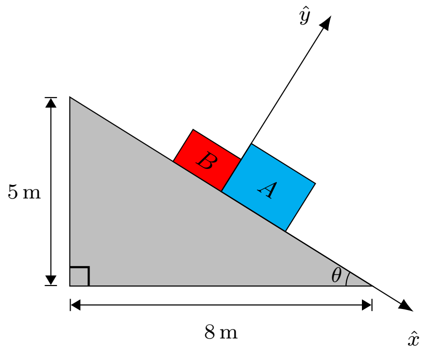
$$\sum F_y = N_A - m_A g\cos\theta = 0 \implies N_A = m_A g\cos\theta$$
$$\sum F_x = -\mu_A N_A + F + m_A g\sin\theta = m_A a$$

(1 point for assumption of same acceleration, 1 point for free body diagram or defined coordinates system, 1 point for each correct expression (4 points in total), 1 point for correct components of gravitational force and 1 point for correct friction force)

Solving for the force F we get:

$$F = \frac{m_A m_B}{m_A + m_B} g(\mu_A - \mu_B)\cos\theta \approx 31.23\,\mathrm{N}$$

(1 point for correct expression and 1 point for correct value).



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Question 3. Mid-air collision of golf balls **(10 points)**

Karl and Gustav play golf on neighboring holes. The golf course is designed in such a way that the fairways of the holes intersect, see Figure 2. Karl hits his golf ball at an angle $\theta_K = 30°$ and an initial velocity v_K. Gustav hits his golf ball at the same time at an angle θ_G and an initial velocity v_G. The golf balls collide at the apex of Gustav's trajectory. **Hint:** The gravitational constant is $g = 9.82\,\mathrm{m\,s^{-2}}$.

(a) Find Gustav's launch angle θ_G and the initial velocities v_K and v_G. (3)

(b) Where does the golf balls land? **Hint:** Assume that the collision is perfectly inelastic, and that the golf balls have the same mass m. (2)

(c) How long are the balls in the air after the collision? (2)

(d) What is the change of kinetic energy due to the collision? (3)

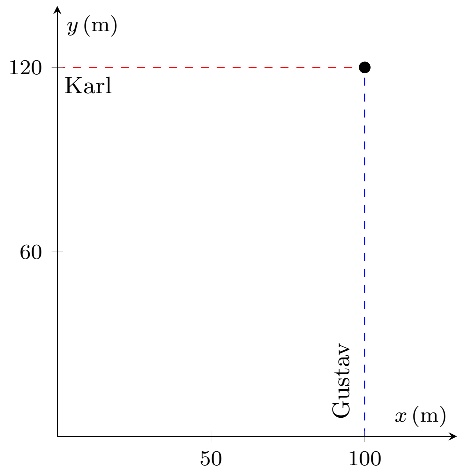

Figure 2: Starting position of Karl and Gustav

Solution:

(a) Projectiles at same height at same time

$$v_K \sin\theta_K = v_G \sin\theta_G$$

Time to apex

$$\tau = \frac{v_G \sin\theta_G}{g} = \frac{v_K \sin\theta_K}{g}$$

Position of impact

$$x_\tau = v_K \cos\theta_K \tau = 100\,\mathrm{m}$$
$$y_\tau = v_G \cos\theta_G \tau = 120\,\mathrm{m}$$

hence

$$v_K = \sqrt{\frac{x_\tau g}{\sin\theta_K \cos\theta_K}} \approx 47.6\,\mathrm{m\,s^{-1}},$$
$$\tau = \frac{v_K \sin\theta_K}{g} \approx 2.4\,\mathrm{s},$$
$$\theta_G = \arctan\frac{\tau^2 g}{y_\tau} \approx 25.7°$$

and

$$v_G = \frac{v_K \sin\theta_K}{\sin\theta_G} \approx 54.9\,\mathrm{m\,s^{-1}}$$

(2 points for correct expression v_K, v_G, θ_G, 1 point for correct values. For only one correct expression give 1 point.)

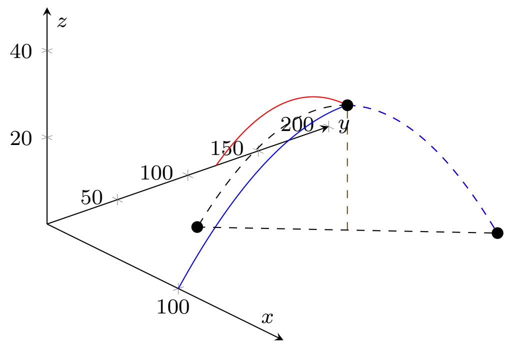

Figure 3: The trajectories of the golf balls and the center of mass

(b) Since the collision is perfectly inelastic the balls stick together and since the total linear momentum is conserved the velocity of the mass center ($\mathbf{v} = \frac{\mathbf{v}_G + \mathbf{v}_K}{2}$) is constant (1 point) hence

$$(x, y)_{\text{final}} = (150\,\mathrm{m}, 180\,\mathrm{m})$$

(1 point for correct answer).

(c) No vertical velocity so same amount of time in air as the time to the apex (1 point)

$$\tau \approx 2.4\,\mathrm{s}$$

(1 point for correct answer)

(d) The change of kinetic energy is

$$\begin{aligned}\Delta E_k &= \frac{m(v_K\cos\theta_K)^2 + m(v_G\cos\theta_G)^2}{2} - \frac{2m}{2}\left(\frac{v_K\cos\theta_K + v_G\cos\theta_G}{2}\right)^2 \\ &= \frac{m}{4}(v_K\cos\theta_K - v_G\cos\theta_G)^2 \approx 17m\,\mathrm{J}\end{aligned}$$

(1 point for correct expression $\Delta E_k = \frac{m\mathbf{v}_G^2}{2} + \frac{m\mathbf{v}_K^2}{2} - \frac{2m\mathbf{v}_{\text{center of mass}}^2}{2}$, 1 point for correct expressions $\mathbf{v}_G, \mathbf{v}_K, \mathbf{v}_{\text{center of mass}}$ and 1 point for correct final expression).

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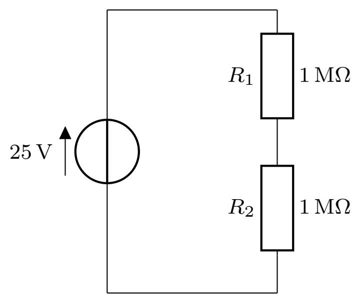
Question 4. Non-ideal electrical sources and meters **(10 points)**

(a) The voltage across the terminals of a black box is measured to $U_m = 25\,\mathrm{V}$ and the current to $I_m = 7.5\,\mathrm{mA}$. The measurements are made with an ideal voltmeter and an ideal ampere meter. The black box can be modeled, equivalently, as either a voltage source with voltage U_T and internal resistance R_T or as a current source with current I_N and internal resistance R_N. Calculate U_T, R_T, I_N, R_N and draw the equivalent circuits. (4)

(b) The voltage over the resistor R_2 is measured with a non-ideal voltmeter with internal resistance $R_{\text{in}} = 10\,\mathrm{M\Omega}$. What voltage does the voltmeter display? (2)



(c) A voltmeter with internal resistance $R_{\text{in}} = 100\,\mathrm{k\Omega}$ can measure voltages up to $U_{\max} = 2\,\mathrm{V}$. Suggest a measuring technique that allows voltage measurements up to $230\,\mathrm{V}$. (2)

(d) An ampere meter with internal resistance $R_{\text{in}} = 10\,\Omega$ can measure current up to $I_{\max} = 20\,\mathrm{mA}$. Suggest a measuring technique that allows current measurements up to $1\,\mathrm{A}$. (2)

Solution:

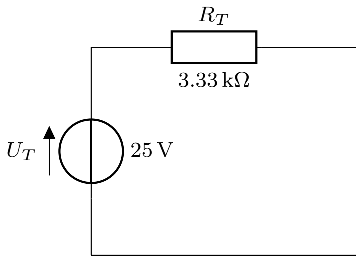
(a) There is does not run any current through an ideal voltmeter and for an ideal ampere meter there will be no voltage drop over it.

Voltage source

For a non-ideal voltage source there will be some internal resistance, i.e there will be some voltage drop inside the voltage source. This can be model by an resistor in series with an ideal voltage source, thus there will be voltage division between any load connected and the internal resistance of the voltage source.

When measuring the the voltage of the black box using an ideal voltmeter the current running through the voltmeter will be zero and therefore the current in the resistor R_T will be zero. Thus the voltage over R_T is zero and we have that $U_T = U_m = 25\,\mathrm{V}$.

Now we measure the current of the black box using an ideal ampere meter, the voltage drop over the ampere meter will be zero and thus the voltage over the resistor R_T will be $U_T = U_m$. To calculate the resistance R_T we can simply use ohms law to obtain $R_T = \frac{U_T}{I_m} = \frac{U_m}{I_m} \approx 3.33\,\mathrm{k\Omega}$.

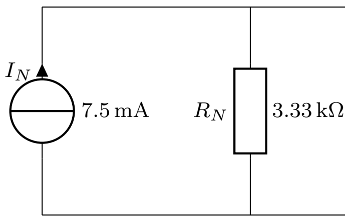


Current source

For a non-ideal current source there will similarly be internal resistance causing a drop in the current compared to an ideal current source, i.e we need a current divider to obtain this. A current divider is simply a resistor in parallel with the load.

Now to calculate I_N and R_N. We can begin by measuring the current over the black box using an ideal ampere meter. We now that there will be no voltage drop over the ideal ampere meter and therefore the voltage over the resistor R_N will be zero and the current through the resistor will thus also be zero. This means that $I_N = I_m = 7.5\,\mathrm{mA}$.

To calculate R_N we instead measure the voltage over the black box using an ideal voltmeter. An ideal voltmeter has no current running through it, thus the current over the resistor R_N will be $I_N = I_m$ and the resistance is thus calculated to $R_N = \frac{U_m}{I_m} = 3.33\,\text{k}\Omega$.



(1 point for each correct circuit diagram and 1 point for each correct pair $((R_T, U_T)$ and $(R_N, I_N))$).

(b) Non-ideal voltmeter can be viewed as a ideal voltmeter in parallel with a resistance R_{in} (Non-zero current through the voltmeter). Voltage divider

$$U_m = 25\,\text{V} \cdot \frac{R_2 \parallel R_{\text{in}}}{R_1 + R_2 \parallel R_{\text{in}}} = 25\,\text{V} \cdot \frac{\frac{10}{11}}{1 + \frac{10}{11}} = 25\,\text{V} \cdot \frac{10}{21} \approx 11.9\,\text{V}$$

(1 point for correct expression and 1 point for correct values).

(c) Known internal resistance $R_{\text{in}} = 100\,\text{k}\Omega$ and max voltage $U_m = 2\,\text{V}$. To allow measurements voltage up to $U'_m = 230\,\text{V}$ place resistor in series with the voltmeter. The resistor R_x and the internal resistance creates a voltage divider

$$U_m = U_o \frac{R_{\text{in}}}{R_x + R_{\text{in}}} = U'_m \frac{R_{\text{in}}}{R_x + R_{\text{in}}} \qquad \text{(1 point)}$$

hence

$$R_x = R_{\text{in}} \left(\frac{U'_m}{U_m} - 1 \right) = 100\,\text{k}\Omega \left(\frac{230\,\text{V}}{2\,\text{V}} - 1 \right) = 11.4\,\text{M}\Omega$$

(1 point for correct value).

(d) Place a resistor R_y in parallel with the ampere meter, which creates a current divider

$$I_m = I_o \frac{R_y}{R_y + R_m} = I'_m \frac{R_y}{R_y + R_m} \qquad \text{(1 point)}$$

hence

$$R_y = \frac{I_m R_{\text{in}}}{I'_m - I_m} = \frac{20\,\text{mA} \cdot 10\,\Omega}{1\,\text{A} - 20\,\text{mA}} = \frac{1}{49}\Omega \approx 200\,\text{m}\Omega$$

(1 point for correct value).