The 37th Science Olympiad for Young Mathematicians, Physicists and Chemists

January 2025
Rudbeckianska upper secondary school
Västerås, Sweden

Physics - Form 10

Question 1. Density of an unknown liquid (10 points)

A spherical shell made of acrylic glass with density $\rho_s = 1.2 \times 10^3\,\mathrm{kg\,m^{-3}}$ contains an unknown liquid. The sphere floats in a container of water at a depth of $h = \frac{3}{2}r_2$. The density of water is $\rho_w = 1 \times 10^3\,\mathrm{kg\,m^{-3}}$ and the internal radius $r_1 = \frac{3}{4}r_2$ where r_2 is the radius of the outer shell, see Figure 1.

(a) Calculate the density ρ_l of the liquid inside the sphere. **Hint:** The volume of a spherical cap of height h is $V = \frac{\pi h^2}{3}(3r - h)$. (6)

(b) Calculate the lowest density of the liquid in the container ρ_c needed for the sphere not to sink. (4)

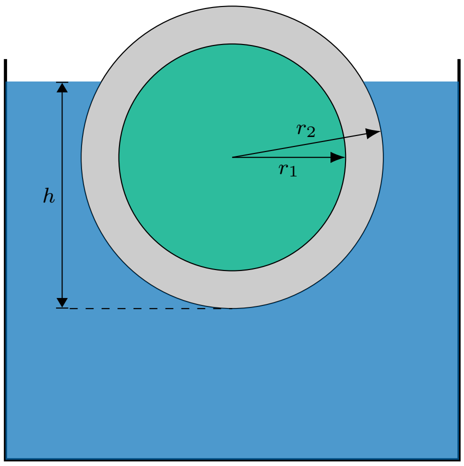

Figure 1: A spherical shell floating in water

Solution:

(a) The total volume of the sphere is

$$V_{\text{tot}} = \frac{4}{3}\pi r_2^3,$$

the volume of inner the sphere is

$$V_l = \frac{4}{3}\pi r_1^3 = \pi r_2^3 \left(\frac{3}{4}\right)^2$$

and the volume of the outer shell is

$$V_s = V_{\text{tot}} - V_l = \pi r_2^3 \left[\frac{4}{3} - \left(\frac{3}{4}\right)^2\right],$$

and the volume of the displaced water (1 point) is

$$V_d = \frac{\pi}{3} h^2 (3r - h) = \frac{\pi r_2^3}{3} \left(\frac{3}{2}\right)^3.$$

(2 points for correct volumes V_tot, V_l, V_s, V_d or 1 point for 3 correct volumes).

Equilibrium of forces (1 point for assumption) so $F_b = V_d \rho_w g = F_g = m_{\text{tot}} g = V_{\text{tot}} \rho_{\text{avg}} g$ hence the average density of the sphere (1 point)

$$\rho_{\text{avg}} = \frac{m_{\text{tot}}}{V_{\text{tot}}} = \frac{V_l \rho_l + V_s \rho_s}{V_{\text{tot}}} = \frac{V_d \rho_w}{V_{\text{tot}}}$$

and

$$\rho_l = \frac{V_d \rho_w - V_s \rho_s}{V_l} = \frac{\frac{\pi r_2^3}{3}\left(\frac{3}{2}\right)^3 - 1.2\pi r_2^3 \left[\frac{4}{3} - \left(\frac{3}{4}\right)^2\right]}{\pi r_2^3 \left(\frac{3}{4}\right)^2} = 2 - 1.2\left[\left(\frac{4}{3}\right)^3 - 1\right] \approx 0.36 \times 10^3 \,\mathrm{kg\,m^{-3}}$$

(1 point for correct expression for ρ_l and 1 point for correct value).

(b) Lowest density when $V_d = V_{\text{tot}}$ (1 point) and equilibrium of forces (1 point) hence

$$\rho_c = \rho_{\text{avg}} = \frac{0.36\pi r_2^3 \left(\frac{3}{4}\right)^2 + 1.2\pi r_2^3 \left[\frac{4}{3} - \left(\frac{3}{4}\right)^2\right]}{\pi r_2^3 \frac{4}{3}} = 0.36\left(\frac{3}{4}\right)^3 + 1.2\left[1 - \left(\frac{3}{4}\right)^3\right]$$

$$\approx 0.84 \times 10^3 \,\mathrm{kg\,m^{-3}}.$$

(1 point for $\rho_c = \rho_{\text{avg}}$ and 1 point for correct value).

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Question 2. Isochoric process of hydrogen gas **(10 points)**

A container is filled with hydrogen gas H_2. The initial pressure of the gas is $200\,\mathrm{kPa}$ and the temperature is $600\,\mathrm{K}$. The hydrogen gas is then cooled to a temperature of $375\,\mathrm{K}$. The container can be considered rigid with a volume of $5\,\mathrm{m}^3$.

(a) Calculate the pressure in the tank after the gas has cooled down? (4)

(b) Calculate the amount of heat transfer to the gas? (6)

Hint: The universal gas constant is $R = 4.1\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$ and the specific heat capacity of hydrogen is taken to $c_v = 10.2\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$.

Solution:

(a) The ideal gas law $P_1V = mRT_1$, which give us that $mR = \frac{P_1V}{T_1}$. Similarly, after the gas has been cooled, $P_2 = \frac{mRT_2}{V} = \frac{P_1T_2}{T_1} = 125\,\mathrm{kPa}$ (2 point for $\frac{P}{T}$ constant, 1 point for expression for P_2 and 1 point for correct value).

(b) The only transfer of energy in this system is the heat transfer (no pressure-volume work due to no change in volume). The heat transfer is given by $Q = mc_v\Delta T$ (1 point), from the ideal gas law $m = \frac{P_1V}{RT_1} \approx 0.41\,\mathrm{kg}$ (2 points). This gives that the heat transfer is $Q = mc_v(T_2 - T_1) \approx -933\,\mathrm{kJ}$ (1 point for correct values). Notice the negative heat transfer, this just means that heat is transferred away from the hydrogen gas (2 points).

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Question 3. Mid-air collision of golf balls **(10 points)**

Karl and Gustav are playing golf on neighboring holes. The golf course is designed in such a way that the fairways of the holes intersect, see Figure 2. Karl hits his golf ball at an angle $\theta_K = 30°$ and an initial velocity v_K. Gustav hits his golf ball at the same time as Karl at an angle θ_G and an initial velocity v_G. The golf balls collide at the apex of Gustav's trajectory, find Gustav's launch angle θ_G and the initial velocities v_K and v_G. **Hint:** The gravitational constant is $g = 9.82\,\mathrm{m\,s^{-2}}$.

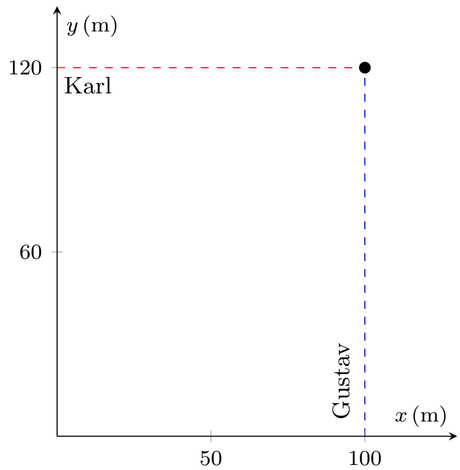


Figure 2: Starting position of Karl and Gustav

Solution:

Projectiles at same height at same time (1 point)

$$v_K \sin\theta_K = v_G \sin\theta_G$$

Time to apex (1 point)

$$\tau = \frac{v_G \sin\theta_G}{g} = \frac{v_K \sin\theta_K}{g}$$

Position of impact (1 point)

$$x_\tau = v_K \cos\theta_K \tau = 100\,\mathrm{m}$$
$$y_\tau = v_G \cos\theta_G \tau = 120\,\mathrm{m}$$

hence

$$v_K = \sqrt{\frac{x_\tau g}{\sin\theta_K \cos\theta_K}} = 47.6\,\mathrm{m\,s^{-1}},$$

$$\tau = \frac{v_K \sin\theta_K}{g} = 2.4\,\mathrm{s},$$

$$\theta_G = \arctan\frac{\tau^2 g}{y_\tau} = 25.7°$$

and

$$v_G = \frac{v_K \sin\theta_K}{\sin\theta_G} = 54.9\,\mathrm{m\,s^{-1}}$$

(1 point for each correct expression v_K, v_G, θ_G, 1 point for each correct value v_K, v_G, θ_G + 1 point if all values are correct)

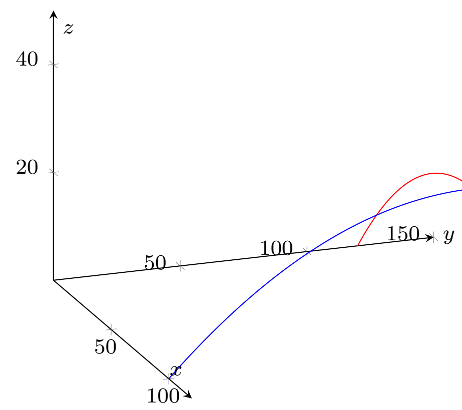
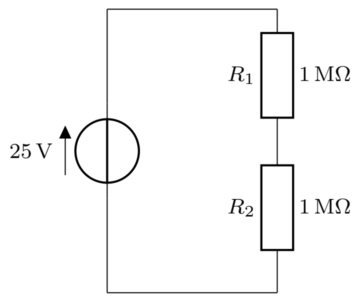


Figure 3: The trajectories of the golf balls

Question 4. Non-ideal electrical meters **(10 points)**

(a) The voltage over the resistor R_2 is measured with a non-ideal voltmeter with internal resistance $R_{\text{in}} = 10\,\text{M}\Omega$. What voltage does the voltmeter display? (2)



(b) A voltmeter with internal resistance $R_{\text{in}} = 100\,\text{k}\Omega$ can measure voltages up to $U_{\max} = 2\,\text{V}$. Suggest a measuring technique that allows voltage measurements up to $230\,\text{V}$. (4)

(c) An ampere meter with internal resistance $R_{\text{in}} = 10\,\Omega$ can measure current up to $I_{\max} = 20\,\text{mA}$. Suggest a measuring technique that allows current measurements up to $1\,\text{A}$. (4)

Solution:

(a) Non-ideal voltmeter can be viewed as a ideal voltmeter in parallel with a resistance R_{in} (1 point) (Non-zero current through the voltmeter). Voltage divider

$$U_m = 25\,\text{V} \cdot \frac{R_2 \parallel R_{\text{in}}}{R_1 + R_2 \parallel R_{\text{in}}} = 25\,\text{V} \cdot \frac{\frac{10}{11}}{1 + \frac{10}{11}} = 25\,\text{V} \cdot \frac{10}{21} \approx 11.9\,\text{V}$$

(1 point for correct answer)

(b) Known internal resistance $R_{\text{in}} = 100\,\text{k}\Omega$ and max voltage $U_m = 2\,\text{V}$. To allow measurements voltage up to $U'_m = 230\,\text{V}$ place resistor in series with the voltmeter (1 point). The resistor R_x and the internal resistance creates a voltage divider (1 point)

$$U_m = U_o \frac{R_{\text{in}}}{R_x + R_{\text{in}}} = U'_m \frac{R_{\text{in}}}{R_x + R_{\text{in}}}$$

hence

$$R_x = R_{\text{in}} \left(\frac{U'_m}{U_m} - 1 \right) = 100\,\text{k}\Omega \left(\frac{230\,\text{V}}{2\,\text{V}} - 1 \right) = 11.4\,\text{M}\Omega$$

(1 point for correct expression R_x and 1 point for correct value).

(c) Place a resistor R_y in parallel with the ampere meter (1 point), which creates a current divider (1 point)

$$I_m = I_o \frac{R_y}{R_y + R_m} = I'_m \frac{R_y}{R_y + R_m}$$

hence

$$R_y = \frac{I_m R_{\text{in}}}{I'_m - I_m} = \frac{20\,\text{mA} \cdot 10\,\Omega}{1\,\text{A} - 20\,\text{mA}} = \frac{1}{49}\Omega \approx 200\,\text{m}\Omega$$

(1 point for correct expression R_y and 1 point for correct value).