The 37$^{\text{th}}$ Science Olympiad for Young Mathematicians, Physicists and Chemists

January 2025
Rudbeckianska upper secondary school
Västerås, Sweden

Physics - Form 12

Question 1. Electron in cathode-ray tube **(10 points)**

The earliest known version of the cathode-ray tube is the Braun tube invented by Fredinand Braun as early as 1897. TV displays used the Braun tube to display images up until the early 2000s when it was replaced with LCD screens, see Figure 1. The cathode-ray tube consists of an electron gun and a deflector which guides the electron to the right position on the screen. In this problem we will consider a cathode-ray tube that uses two electrically charged plates to generate an electric field. The position where the electron hits the screen can be chosen by changing the electric field.

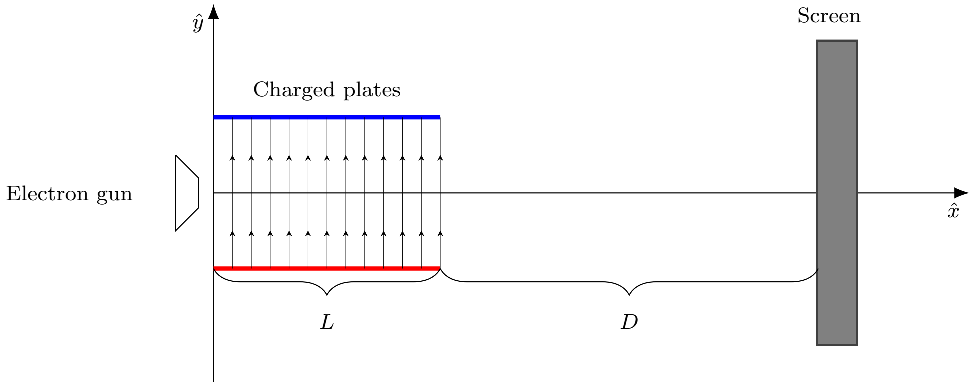

Figure 1: The cross section of the cathode-ray tube.

(a) In a cathode-ray tube, an electron of mass m_e is projected with velocity $\mathbf{v} = v_0\hat{x}$. The charged plates generate a constant electric field of $\mathbf{E} = E_0\hat{y}$. Where does the electron hit the screen? **Hint:** You should find the expression of the position in terms of the variables E_0, m_e, v_0, L and D. Neglect any other external forces and assume that the electron is at the origin at time t_0. (4)

(b) Determine where the electron will hit the screen if instead we have a changing electric field given by $\mathbf{E} = E_0 \sin(\omega t)\hat{y}$. Here ω is the angular frequency of the electric field and the electron enters the electric field at time $t_0 = 0$. **Hint:** You should find the expression of the position in terms of the variables E_0, m_e, v_0, L, D and ω. (6)

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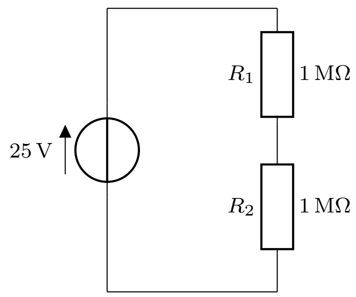
Question 2. Non-ideal electrical sources and meters **(10 points)**

(a) The voltage across the terminals of a black box is measured to $U_m = 25\,\mathrm{V}$ and the current to $I_m = 7.5\,\mathrm{mA}$. The measurements are made with an ideal voltmeter and an ideal ampere meter. The black box can be modeled, equivalently, as either a voltage source with voltage U_T and internal resistance R_T or as a current source with current I_N and internal resistance R_N. Calculate U_T, R_T, I_N, R_N and draw the equivalent circuits. (2)

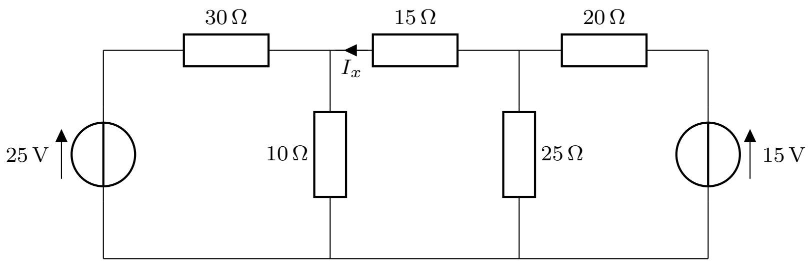
(b) The voltage over the resistor R_2 is measured with a non-ideal voltmeter with internal resistance $R_{\text{in}} = 10\,\mathrm{M\Omega}$. What voltage does the voltmeter display? (2)



(c) A voltmeter with internal resistance $R_{\text{in}} = 100\,\mathrm{k\Omega}$ can measure voltages up to $U_{\max} = 2\,\mathrm{V}$. Suggest a measuring technique that allows voltage measurements up to $230\,\mathrm{V}$. (2)

(d) An ampere meter with internal resistance $R_{\text{in}} = 10\,\Omega$ can measure current up to $I_{\max} = 20\,\mathrm{mA}$. Suggest a measuring technique that allows current measurements up to $1\,\mathrm{A}$. (2)

(e) Calculate the current I_x (2)



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Question 3. Flood protection of Västerås **(10 points)**

During the summer of 2023 Svartån, the river that runs through Västerås, was flooded. During the autumn of the same year Västerås bought 200 meters of flood barriers. The city of Västerås wants to understand when and how these barriers might fail. Therefore, your task is to calculate the critical height of the water at which the barrier fails. The possible points of failure considered are that the barrier can either start to slide or tip.

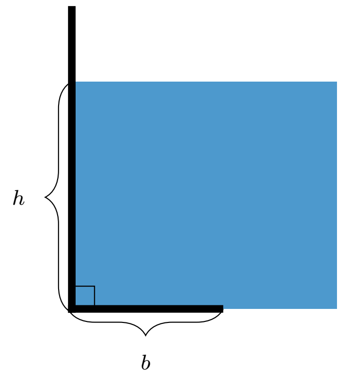

Figure 2: The cross section of the flood barrier.

A cross section of the flood barrier can be seen in Figure 2 and it has a total length of L. The coefficient of friction between the ground and the barrier can be assumed to be $\mu = 0.8$. Assume that the weight of the barrier is negligible in relation to the weight of the water and that there is a perfect seal between the ground and the barrier until it begins to tip. Also, assume that the barrier is a rigid body.

(a) Calculate the critical height of the water when the barrier starts to slide, that is, the relation between height h and length b? (4)

(b) Calculate the critical height of the water when the barrier starts to tip, that is, the relation between height h and length b? (4)

(c) How will the barrier fail and what is the critical height of the barrier? (2)

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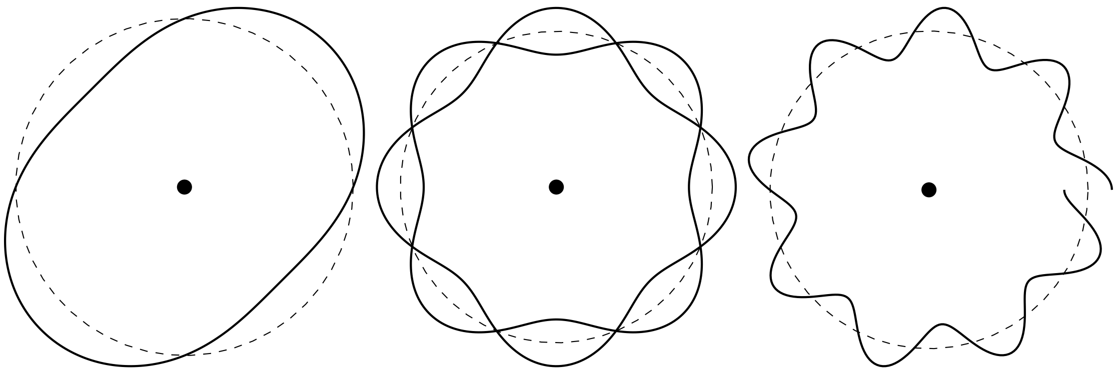
Question 4. Wave-duality of particles and the quantization of hydrogen atoms (10 points)

De Broglie postulated in 1924 the wave-duality of matter and suggested that all particles could be considered waves with wavelength

$$\lambda = \frac{h}{p}$$

where p is the linear momentum of the particle. He suggested that the electron's orbit around the nucleus could be thought of as standing waves, see Figure 3.



(a) Standing wave

(b) Standing wave with lower wavelength than (a)

(c) Destructive interference

Figure 3: Standing stationary waves and destructive interference

In 1913 Bohr postulated that electrons orbit the nucleus in stable circular orbits and that the angular momentum of electrons are quantized (discrete/non-continuous values). De Broglie's condition of standing waves can be seen as an interpretation of Bohr's original postulate.

Neglect any gravitational effects and assume that the mass of the nucleus is much larger than that of the electron ($m_p \gg m_e$) such that the nucleus is stationary. The electron can be treated as a particle orbiting the nucleus; however, when viewed as a wave, the electron must still fulfill the condition of a standing wave. The charge of the nucleus is $q_p = e$ and the charge of the electron $q_e = -e$.

(a) Show that de Broglie's condition of standing waves is equivalent to Bohr's original postulate of quantized angular momentum. (2)

(b) Show that the condition of standing waves leads to quantized radii of the electron orbits and derive an expression for r_n. (4)

(c) Derive an expression for the energy levels E_n corresponding to the quantized radii. (4)

Hint: The reduced Planck's constant $\hbar = \frac{h}{2\pi}$ might be useful to simplify notation.