The 36th International Science Olympiad

for Young Mathematicians, Physicists and Chemists

MATHEMATICS

GRADE 10

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Munkkiniemen yhteiskoulu,
Helsinki Finland

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All answers must be justified with careful and correct reasoning.
All questions are worth the same value.
Calculators are not permitted.

1. Four children, Andre, Antti, Alise, and Amelija, were playing outside my house when suddenly I heard a loud noise followed by much shouting. I rushed out to find that one of the children had broken the window with a football. Andre and Antti spoke almost at the same time: Andre saying, "It wasn't me!" Antti saying, "It was Amelija!" Alise yelled, "No, it was Antti!" Amelija said, with a straight face, "Antti is a liar." Only one of them was telling the truth, so who broke my window?

2. Suppose Aalto University has 2500 lockers and 2500 students, both labeled from 1 to 2500. They tried the following experiment. All lockers are initially closed. Then student number 1 opens all the lockers. Student number 2 closes the even numbered lockers. Student number 3 changes the status (that is, open to closed or closed to open) of all lockers that are multiples of 3. This continues with each student changing the status of all the lockers which are numbered by multiples of their number. After 700 students experimented, what is the status of locker number 625?

3. Which one of the following numbers is the largest: $A = 200^{2000}, B = 9^{5120}, C = 2^{(2^{14})}$, or $D = (64000) \cdot (64001) \cdot (64002) \cdot (64003) \cdot (64004) \cdots (65000)$ (that is, the product of the consectuitve integers from 64000 to 65000).

4. Two congruent non-overlapping (edges may touch) equilateral triangles are placed wholly within a square of side length 1. What is the maximum area they could cover? (*Congruent* = same shape and size. *Equilateral triangle* = traingle with all three sides the same length. The triangles and the squares may also touch on their edges.)

5. Let a, b, and c be positive integers such that $a \leq b \leq c$. We have abc identical wooden cubes, and use them to build an $a \times b \times c$ block (rectangular cuboid). Suppose the number of cubes with at least one external face is equal to the number of totally enclosed cubes. If a = 8, what is the largest possible value of abc?