The 34th International Science Olympiad for Young Mathematicians, Physicists and Chemists
November 30, 2021
Physics – Form 10

1.

a) Let the car's top speed be v_m. The amount of time it takes to accelerate is $\Delta t_1 = v_m/a_1$ (*1 point*). Therefore, it covers a distance of $\Delta L = a_1 \Delta t_1^2/2 = v_m^2/(2a_1)$ when accelerating (*1 point*). During the remaining time in the first lap, the car covers a distance of $v_m(t_1 - \Delta t_1) = L - \Delta L$ (*1 point*), where we also used the fact that the lap is $L = 1000\,\mathrm{m}$ long. Rearranging and expanding the terms, we get

$$v_m^2/(2a_1) - v_1 t_1 + L = 0. \quad \textit{(1 point)}$$

This is a quadratic equation whose solutions are

$$v_m = \frac{t_1 \pm \sqrt{t_1^2 - 2L/a_1}}{1/a_1} = 286{,}6\,\mathrm{m/s}, 55{,}8\,\mathrm{m/s}.$$

It's clear that the bigger solution is not physical because it would involve the car accelerating for $(286{,}6/8)\mathrm{s} = 35{,}8\,\mathrm{s}$ which is bigger than the duration of the first lap (*1 point*). Hence, the car's top speed is $v_m = 55{,}8\,\mathrm{m/s} \approx 200\,\mathrm{km/h}$ (*1 point*).

b) The driver's fastest lap occurs when it's driving at top speed (*1 point*). This gives the fastest lap time to be $L/v_m = 18\,\mathrm{s}$ (*1 point*).

c) The last lap is the same as the first one, only that the car is decelerating with $a_2 = 11\,\mathrm{m/s^2}$. Hence we get, similarly to the first part,

$$v_m^2/(2a_2) - v_m t_2 + L = 0, \quad \textit{(1 point)}$$

with t_2 being the duration of the last lap. From here,

$$t_2 = \frac{v_m^2/(2a_2) + L}{v_m} = 20{,}4\,\mathrm{s} \approx 20\,\mathrm{s}. \quad \textit{(1 point)}$$

2.

a) Let the mass of the aluminium bar be m, and its temperature T_a. After thermal equilibrium has been reached in both cases, both the water and the amount of aluminium that's in water all assume uniform temperature (*1 point*). Notably, the aluminium changes the total heat capacity of the system (*1 point*). Applying conservation of energy in both situations, we get

$$Mc_wT_1 + mc_aT_a = (Mc_w + mc_a)T_2, \tag{1}$$
$$Mc_wT_1 + 2mc_aT_a = (Mc_w + 2mc_c)T_3. \quad \textit{(4 points)} \tag{2}$$

The two equations can be solved for m and T_a. Take $2 \cdot (1) - (2)$, this yields

$$Mc_wT_1 = Mc_w(2T_2 - T_3) + 2mc_a(T_2 - T_3)$$

and so

$$m = M\frac{c_w}{2c_a}\frac{2T_2 - T_1 - T_3}{T_3 - T_2} = 330\,\mathrm{g}. \quad \textit{(2 points)}$$

b) T_a can be found by simply plugging m either into (1) or (2). This yields

$$T_a = T_2 + \frac{M}{m}\frac{c_w}{c_a}(T_2 - T_1) = 500\,^\circ\mathrm{C}. \quad \textit{(2 points)}$$

3.

a) Let's denote the density of the unknown liquid as ρ. Because both branches of the tube are connected, the equilibrium pressures at the bottom of both tubes must be equal (*2 points*). Indeed, if they weren't equal, the pressure differential would drive a water flow from one branch of the tube to the other, meaning the system wasn't in equilibrium. We can write this condition down for both branches, relating the unknown liquid's density with the known change in height of the left branch. Further, we can use conservation of the amount of water (*2 point*).

After the unknown liquid has been poured, the water level in the left branch is $H - h + \Delta h$ and this gives a pressure of $\rho_w g(H - h + \Delta h)$ at the bottom of the tube (*1 point*). The right branch has water to the height of $H - h - \delta h$ (from conservation of water) (*1 point*), leading to a pressure of $\rho_w g(H - h + \Delta h) + V\rho g/S$, where $V\rho g/S$ is the pressure contribution from the unknown liquid (*1 point*). Because the two pressures are equal, we find

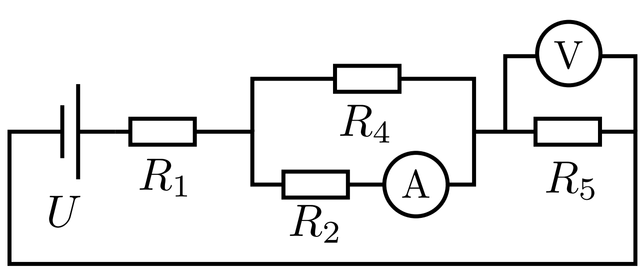
$$\rho = \rho_w \frac{2\Delta h S}{V} = 800\,\mathrm{kg/m^3}. \quad \textit{(1 point)}$$

b) Let's denote the further change in the level of the left branch as $\Delta h'$. It's clear that the level of the left branch can be raised the furthest by pouring so much of the unknown liquid such that it almost starts overflowing (*1 point*). We now proceed similar to the previous subtask. The water pressure in the left branch is $\rho_w g(H - h + \Delta h + \Delta h')$ (*1 point*). The right branch has water up to a level of $H - h - \Delta h - \Delta h'$ and thus unknown liquid of height $h + \Delta h + \Delta h'$. The pressure in the right branch is thus $\rho_w g(H - h - \Delta h - \Delta h') + \rho g(h + \Delta h + \Delta h')$ (*1 point*). Equating the pressures, we get

$$\Delta h' = \frac{\rho(h + \Delta h) - 2\rho_w \Delta h}{2\rho_w - \rho} = 3{,}3\,\mathrm{cm}. \quad \textit{(1 point)}$$

4.

a) First note that the ammeter short-circuits R_3 (*1 point*). Hence, we can remove it from our considerations. It also helps to redraw the circuit, shown below. We see that the resistors form a simple sequence of series and parallel connections (*1 point*). The total resistance of the circuit is $R_{\mathrm{tot}} = R_1 + R_5 + \frac{R_4 R_2}{R_2 + R_4} = 8{,}9\,\Omega$ (*1 point*). The total current through R_1 and R_5 is thus $I_{\mathrm{tot}} = U/R_{\mathrm{tot}} = 0{,}90\,\mathrm{A}$ (*1 point*). The current through R_2 is then $I_{\mathrm{tot}} R_4/(R_2 + R_4) = 0{,}56\,\mathrm{A}$ (*2 points*). This also coincides with the reading of the ammeter.



b) The voltmeter reading is simply the voltage on R_5 and so is $I_{\mathrm{tot}} R_5 = 4{,}5\,\mathrm{V}$ (*2 points*).