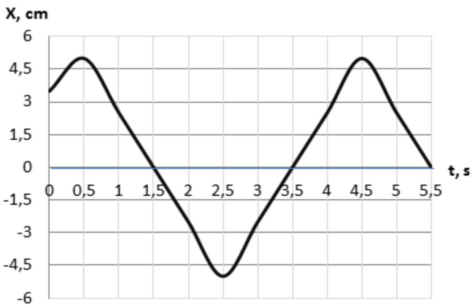
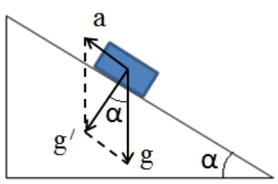
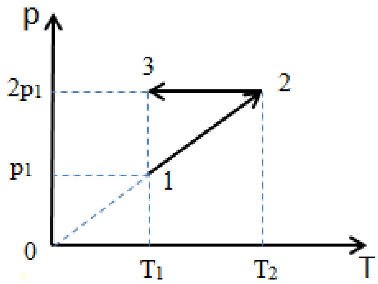
PHYSICS -12

1. The ball hung on the weightless rod (or a thread) oscillates at 0.25 Hz frequency. Amplitude is 5 cm, the initial phase $\varphi_0 = 45°$. Acceleration of gravity $g = 10\ m/s^2$
   a. Calculate how much time will pass since the beginning of oscillation when the ball will be at the maximum distance from the equilibrium position?
   b. Draw the graph of oscillations.
   c. Calculate the length of the pendulum.
   d. The pendulum is in the wagon which moves up on the inclined plane which is inclined at an angle of 40°. Calculate the period of the pendulum.
   e. What is potential energy of the oscillating ball after 1.5 s from the beginning of oscillation?

Solution:

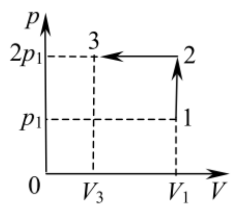
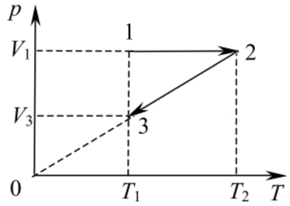
a) $x = A\sin(\omega t + \varphi_0)$, $x = A$ $\frac{x}{A} = \sin(\omega t + \varphi_0)$, $1 = \sin(\omega t + \varphi_0)$ $(\omega t + \varphi_0) = \arcsin 1$, $\arcsin 1 = \frac{\pi}{2}$, $\varphi_0 = \frac{\pi}{4}$, $\omega = 2\pi\nu = \frac{\pi}{2}\ rad/s$ $\frac{\pi}{2}t + \frac{\pi}{4} = \frac{\pi}{2}$ $t = \frac{1}{2} = 0.5\ s$	**1 mark** **1 mark**
b) $x_0 = A\sin(\omega t + \varphi_0)$ $t_0 = 0$ $x_0 = A\sin(\omega t + \varphi_0)$ $x_0 = A\sin\varphi_0$  $x_0 = 0.05\sin 45 = 0.035\ m$	**1 mark** for the correct x_0 **0.5 mark** for the correct amplitude in the graph **0.5 mark** for the correct period in the graph **1 mark** for the correct curve
c) $T = 2\pi\sqrt{\frac{l}{g}}$, $T = \frac{1}{\nu}$ $l = \frac{T^2 g}{4\pi^2} = \frac{g}{4\pi^2\nu^2}$ $l = 4$ m	**1 mark** **1 mark**
d)  $g^{/} = g\cos\alpha$ $T = 2\pi\sqrt{\frac{l}{g\cos\alpha}} = 4.5$ s	**1 mark** **1 mark**
e) $E_P = 0$ *from the graph* or $E_P = \frac{kx^2}{2}$, $x = A\sin\left(\frac{\pi}{2}\cdot\frac{3}{2} + \frac{\pi}{4}\right) = A\sin\pi = 0$ $E_P = 0$	**1 mark** **or** **1 mark**

2. Figure shows the p-T diagram of 3 moles of monatomic ideal gas. The initial gas temperature is 400 K. R = 8.31 J/(mol·K), $N_A = 6\cdot10^{23}$ mol^{-1}.

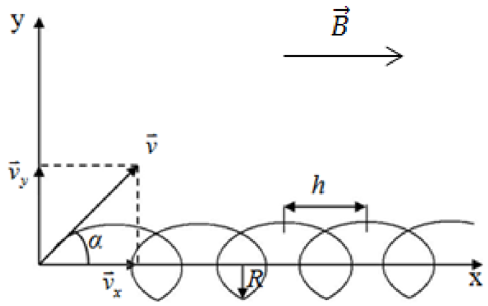


a. Draw a diagram on p-V axes.
b. Draw a diagram on V-T axes.
c. Calculate the change in internal energy of the gas.
d. Calculate the work done by the gas.
e. What quantity of heat was given to the gas?
f. How many gas molecules are under these conditions?

Solution:

a) 	**1 mark** for correct process 1→2 **1 mark** for correct process 2→3
b) 	**1 mark** for correct process 1→2 **1 mark** for correct process 2→3
c) $\Delta U = \frac{3}{2}\nu R\Delta T$ $\Delta T = 0, \quad \Delta U = 0$	**0.5 mark** **0.5 mark**
d) $A = A_{12} + A_{23}$ $A_{12} = 0, \; A = A_{23}$ $A = p\Delta V = 2p_1(V_3 - V_2)$ $V_2 = \frac{\nu RT_1}{p_1}, V_3 = \frac{\nu RT_1}{2p_1}$ $A = 2p_1\left(\frac{\nu RT_1}{2p_1} - \frac{\nu RT_1}{p_1}\right) = -\nu RT_1 = -10kJ$ $A = -10kJ$	**0.5 mark** **0.5 mark** **0.5 mark** **1 mark** **0.5 mark**
e) $Q = \Delta U + A$ $\Delta U = 0, \quad Q = A = -10\ kJ$	**0.5 mark** **0.5 mark**
f) $N = \nu\cdot N_A$ $N = 1.8\cdot10^{24}$	**0.5 mark** **0.5 mark**

3. An electron moves in a magnetic field. Its trajectory is a spiral line, with a radius R = 1 cm and step h = 3 cm. Induction of magnetic field B = 1 mT.
$q_e = -1.6 \cdot 10^{-19}$ C, $m_e = 9.1 \cdot 10^{-31}$ kg.



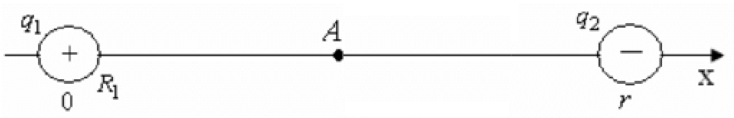
a. Calculate the velocity v_x of the electron.
b. Calculate the velocity v_y of the electron
c. Calculate the total velocity v of the electron.

Solution:

a) $\boldsymbol{v_y}$ $F_L = q_e v_y B \sin\alpha$, $\quad sin\alpha = 90^o$, $\quad F_L = q_e v_y B$	**1 mark**
Applying Newton's second law: $F = m_e a_y$, here a_y – centripetal acceleration: $a_c = \frac{v_y^2}{R}$, $q_e v_y B = m_e \frac{v_y^2}{R}$.	 **2 mark**
$v_y = \frac{q_e BR}{m_e} = 1.8 \cdot 10^6$ m/s.	**1 mark**
b) $\boldsymbol{v_x}$ $h = v_x T$, $\quad v_x = \frac{h}{T}$, here T – the period of rotation of an electron	**1 mark**
$T = \frac{2\pi R}{v_y}$, $T = \frac{2\pi R m_e}{q_e BR} = \frac{2\pi m_e}{q_e B}$	**2 mark**
$v_x = \frac{h q_e B}{2\pi m_e} = 8.4 \cdot 10^5$ m/s.	**1 mark**
c) $\boldsymbol{v}$ $v = \sqrt{v_x^2 + v_y^2}$	**1 mark**
$v = \sqrt{\frac{h^2 q_e{}^2 B^2}{4\pi^2 m_e^2} + \frac{q_e{}^2 B^2 R^2}{m_e^2}} = 1{,}95 \cdot 10^6 \frac{m}{s} \approx 2 \cdot 10^6 \frac{m}{s}$	**1 mark**

4. Two copper balls are at a distance of 20 cm from each other. Their mass is equal to 1 kg each. $k = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 m/F$, $q_e = -1.6 \cdot 10^{-19}$ C, $m_e = 9.1 \cdot 10^{-31}$ kg, $N_A = 6 \cdot 10^{23}$ mol^{-1}, G = $6.67 \cdot 10^{-11}$ Nm^2/kg^2.

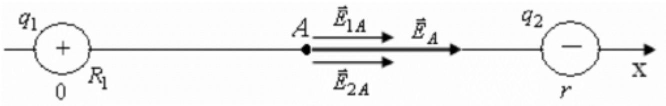
Cu	***Z***	***ρ, kg/m³***	***M, g/mol***
	29	8900	63.5



a. Calculate the electrostatic force between these balls, if only $1 \cdot 10^{-10}$ % part of electrons of the first ball was moved to the second ball.
b. Calculate the potentials of the balls.
c. What strength of the electric field is created by the first ball at its surface?
d. What strength of the electric field will be in the center of the line which is between the balls and connects them? Draw the strengths of the fields and their resultant.
e. How many times the electrostatic force of interaction between two balls is greater than the gravitational force between them?

Solution:

a) The number of electrons in the ball: $N_e = N \cdot Z$, $N = \frac{m}{M} N_A$	**0.5 mark**
Moved electric charge from the first ball to the second ball: $q_2 = q_e n N_e$, $n = 1 \cdot 10^{-10}\,\% = 1 \cdot 10^{-12}$	**0.5 mark**
Electric charge of the first ball: $q_1 = \lvert q_2 \rvert = \frac{q_e n m N_A Z}{M}$	**1 mark**
$F = k\frac{q_1 q_2}{r^2} = k\left(\frac{q_e n m N_A Z}{Mr}\right)^2 = 4.35 \cdot 10^{-4} N$	**1 mark**
b) $\varphi_1 = k\frac{q_1}{R_1}$,	**0.5 mark**
$\rho = \frac{m}{V}$, $V = \frac{4}{3}\pi R_1^3$, $R_1 = \sqrt[3]{\frac{3m}{4\pi\rho}}$	**0.5 mark**
$\varphi_1 = \frac{q_e n m N_A Z}{M\sqrt[3]{\frac{3m}{4\pi\rho}}} = 132$ kV	**1 mark**
c) $E_1 = k\frac{q_1}{R_1^2} = k\frac{q_e n m N_A Z}{M\sqrt[3]{\left(\frac{3m}{4\pi\rho}\right)^2}}$	**0.5 mark**
$E_1 = 4.42 \cdot 10^7 \frac{V}{M}$	**0.5 mark**

d) According to the fields superposition principle: $\overrightarrow{E_A} = \overrightarrow{E_{1A}} + \overrightarrow{E_{2A}}$  $E_A = k\frac{q_1}{r_A^2} + k\frac{q_2}{r_A^2} = k\frac{8q_1}{r^2}$ $r_A = \frac{r}{2}$, $q_1 = q_2$ $E_A = k\frac{8q_1}{r^2} = k\frac{8q_e nmN_A Z}{Mr^2} = 79.1\,\frac{kV}{m}$	**0.5 mark** **1 mark** **0.5 mark** **1 mark**
e) $F_g = G\frac{m_1 m_2}{r^2} = G\frac{m^2}{r^2}$ $\frac{F}{F_g} = \frac{k}{G}\left(\frac{q_e nN_A Z}{M}\right)^2 = 2.6\cdot 10^{11}$	**0.5 mark** **0.5 mark**