SOLUTIONS

Form 10, problem 1

a) Gauss' formula: $\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$, a = 2,0 m, b = 0,22 m. (1 p)

$$\frac{1}{f}=\frac{a+b}{ab}$$

$$= \ f=\frac{ab}{a+b}=\frac{2{,}0\,\mathrm{m}\cdot 0{,}22\,\mathrm{m}}{2{,}0\,\mathrm{m}+0{,}22\,\mathrm{m}} = 0{,}198198\ \mathrm{m} \approx \underline{0{,}20\ \mathrm{m}}$$

(right answer: 1 p)

b) b = 0,25 m (1 p)

$$\frac{1}{a}=\frac{b-f}{bf}$$

$$a=\frac{bf}{b-f}=\frac{0{,}25\,\mathrm{m}\cdot 0{,}198198\,\mathrm{m}}{0{,}25\,\mathrm{m}-0{,}198198\,\mathrm{m}} = 0{,}956522\ \mathrm{m} \approx \underline{0{,}96\ \mathrm{m}}$$

(right answer: 1 p)

c) The objective is made of two similar lenses, 1 and 2. We know that the distance of the lens 2 is $b_2 = b$ = 0,25 m. Distance of the "object" for lens 2 is then $a_2 = a$, which is the distance of the picture from lens 1, $b_1 = -a$.

(understanding physical situation: max 2p)

$$a_1=\frac{b_1 f}{b_1-f}=\frac{-0{,}956522\,\mathrm{m}\cdot 0{,}198198\,\mathrm{m}}{-0{,}956522\,\mathrm{m}-0{,}198198\,\mathrm{m}} = 0{,}164179\ \mathrm{m} \approx \underline{0{,}16\ \mathrm{m}}$$

(right answer: 1 p)

d) Focal distance is the distance from an object that is indefinitely far away from the lens. Thus for lens 1, $b_1 = f$. For lens 2 this is the distance of the object, $a_2 = -f$. Focal length of the combination lens is therefore the distance of the picture from lens 2:

(understanding physical situation: max 2p)

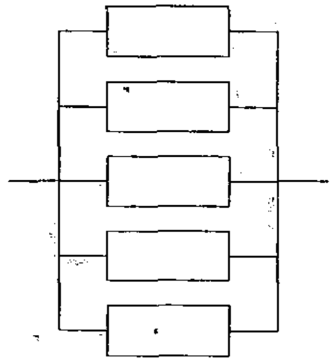
$$f_{1+2}=b_2=\frac{a_2 f}{a_2-f}=\frac{-f\cdot f}{-f-f}=\frac{-f^2}{-2f}=\frac{f}{2} = 0{,}478261\ \mathrm{m} \approx \underline{0{,}48\ \mathrm{m}}$$

~~0,48 m~~ 0,10 m

(right answer: 1 p)

Form 10, problem 2

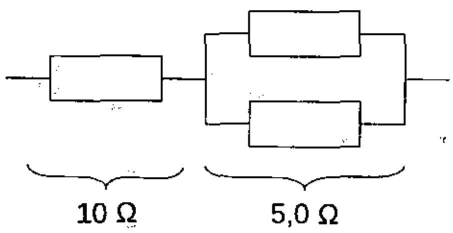
a)



$$R_{tot}=1/\left(\frac{5}{10\,\Omega}\right)=2{,}0\,\Omega$$

(circuit + calculation: 1p)

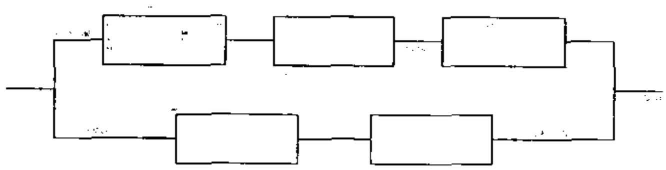
b)



$$R_{tot}=10\,\Omega+1/\left(\frac{2}{10\,\Omega}\right)=15\,\Omega$$

(circuit + calculation: 1p)

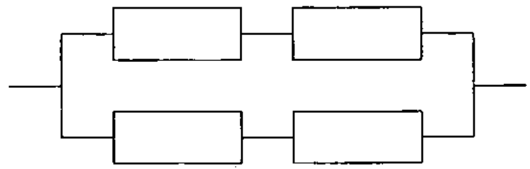
c)



$$R_{tot}=1/\left(\frac{1}{3\cdot 10\,\Omega}+\frac{1}{2\cdot 10\,\Omega}\right)=1/\left(\frac{50\,\Omega}{600\,\Omega^2}\right)=12\,\Omega$$

(circuit: 1p, calculation: 1p)

d)



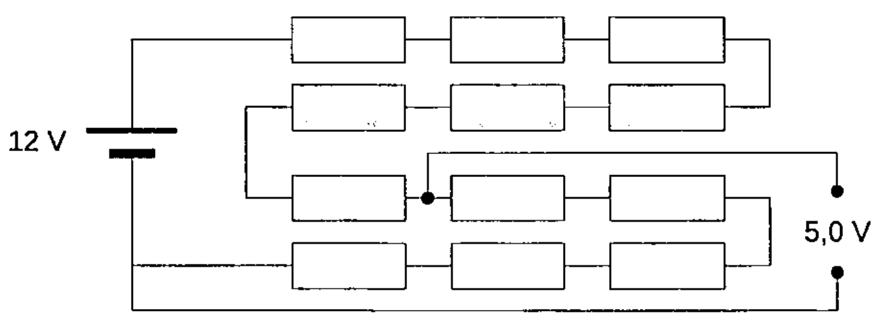
$$R_{tot}=1/\left(2\cdot\frac{1}{2\cdot 10\,\Omega}\right)=10\,\Omega$$

The terminal voltage of every resistor is $\frac{1}{2}U_{tot}$, and the current through every resistor one is $\frac{1}{2}I_{tot}$. Therefore the power for every resistor is $P = \frac{1}{2}U_{tot}\cdot\frac{1}{2}I_{tot} = \frac{1}{4}P_{tot}$. Power is distributed evenly among the resistors, the total maximum power of the circuit is thus 4 W.

(circuit: 1p, resistance calculation: 1p, fiquring out power: 1p)

e) Every circuit that consists of a reasonable number of resistors and does not have too large power dissipation in any resistor is accepted.

Solution 1: 12 resistors

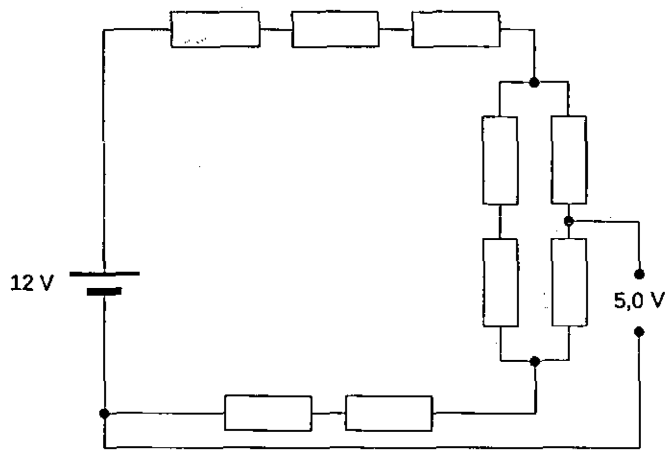


12 similar resistors in series, the voltage drops an equal amount in every resistor. Therefore the voltage across 5 resistors is 5 V.

Power dissipation: $I=\frac{U}{R}=\frac{12\,V}{12\cdot 10\,\Omega}=0{,}1\,\mathrm{A}$, power of one resistor is

$P=RI^2=10\,\Omega\cdot(0{,}1\,A)^2=0{,}1\,\mathrm{W}$ which is below the limit.

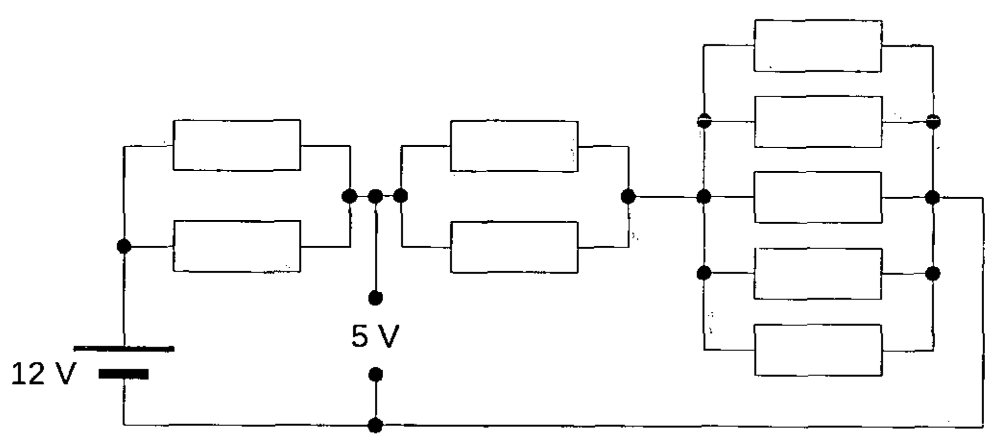
Solution 2: 9 resistors



Let's think first 6 equal resistors in series, for every resistor there is a 2 V drop. One of the resistors is replaced with a circuit of 2 in serial, 2 in parallel resistors, like in d). Output for is 5 V is taken as shown in the picture.

Power dissipation: total current through the circuit is $I=\frac{U}{R}=\frac{12\,V}{6\cdot 10\,\Omega}=0{,}2\,\text{A}$. Power of every resistor in series is $P=RI^2=10\,\Omega\cdot(0{,}2\,A)^2=0{,}4\,\text{W}$. Power of every resistor in parallel is ¼·0,4 W = 0,1 W.

An example circuit that divides the voltage right, but has parts with too large power dissipation:



Total resistance is (5+5+2) Ω = 12 Ω, thus the output shown the has 5 V voltage. Total current is $I=\frac{U}{R}=\frac{12\,V}{12\,\Omega}=1{,}0\,\text{A}$. For example in the two leftmost parallel resistors the current is divided in two, and the power of each resistor is $P=RI^2=10\,\Omega\cdot(0{,}5\,A)^2=2{,}5\,\text{W}$. This is over the 1 W specification, and the circuit may fail.

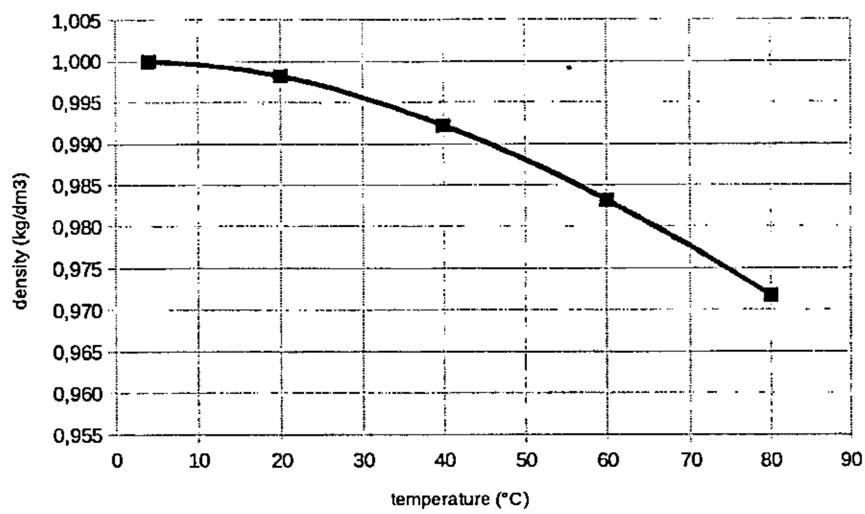
(circuit: 1p, proof of voltage output 1p, fiquring out power: 1p)

Form 10, problem 3

a) The phenomenon is a result of thermal expansion of water. When water expands, its volume increases and density decreases. For a floating ball, the buoyant force is equal with the weight. When temperature rises and density of water decreases, at certain point the buoyant force becomes less than the weight of the ball, and the ball sinks.

(thermal expansion + density: 1p, floating: 1p, change of buoyancy: 1p)

b) Graph $\varrho(t)$ is drawn using the data table. According the graph, at $t = 45°C$ density of water is $\rho_{45C} = 990\ kg/m^3$. The ball floats when it has the mass of equal volume of water.



$$m = \rho_{45C} \cdot \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = 990\ kg/m^3 \cdot \frac{4}{3}\pi\left(\frac{40{,}2 \cdot 10^{-3}\ m}{2}\right)^3 = 0{,}0336753\ kg$$

Answer: mass of the ball is 33,7 g.

(graph: 1p, density: 1p, mass: 1p)

c) $$\rho = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} = \frac{0{,}0336753\ kg + 0{,}00010\ kg}{\frac{4}{3}\pi\left(\frac{34{,}2 \cdot 10^{-3}\ m}{2}\right)^3} = 992{,}93984\ kg/dm^3$$

Reading the graph says that water has this density at 37°C.

(density: 1p, temperature: 1p)

d) The buoyant force is equal with the weight of the air that the balloon displaces. The balloon floats when the total weight of the shell and helium equals with the buoyant force.

$$G_k+G_h=N=m_i g$$
$$(m_k+m_h)g=m_i g$$
$$m_k+m_h=\rho_i V$$
$$m_k+m_h=\rho_i\left(\frac{m_h}{\rho_h}\right)$$
$$m_k=\left(\frac{\rho_i}{\rho_h}-1\right)m_h$$
$$m_h=\frac{m_k}{\frac{\rho_i}{\rho_h}-1}=\frac{3{,}7\,\mathrm{g}}{\frac{1{,}293\,\mathrm{kg/m^3}}{0{,}1787\,\mathrm{kg/m^3}}-1}=0{,}593368\,\mathrm{g}$$

Answer: the balloon must be filled with more than 0,59 g helium.

($m_k+m_h=\rho_i\left(\frac{m_h}{\rho_h}\right)$ 1p, mass of helium 1p)

Form 10, problem 4

a) $v_A = \frac{2{,}0\,\text{m}}{9{,}2\,\text{s}} = 0{,}2173913\,\text{m/s} \approx \underline{0{,}22\,\text{m/s}}$ (1 p)

$a_B = \frac{2s}{t^2} = \frac{2 \cdot 2{,}0\,\text{m}}{(5{,}4\,\text{s})^2} = 0{,}1371742\,\text{m/s}^2 \approx \underline{0{,}14\,\text{m/s}^2}$ (1 p)

b) Cart A: $s_A = v_A t + s_0$, where s_0 is the lead of A.
$s_0 = v_A t_e = 0{,}2173913\,\text{m/s} \cdot 2{,}4\,\text{s} = 0{,}5217391\,\text{m}$ (1p)

Cart B: $s_B = \frac{1}{2} a_B t^2$.

When B passes A, $s_A = s_B$. (1p)

$$v_A t + s_0 = \frac{1}{2} a_B t^2$$
$$\frac{1}{2} a_B t^2 - v_A t - s_0 = 0$$
$$t = \frac{v_A \pm \sqrt{v_A^2 + 2 a_b \cdot s_0}}{a_b}$$
(1p)

$$t = \frac{0{,}2173913\,\text{m/s} \pm \sqrt{(0{,}2173913\,\text{m/s})^2 + 2 \cdot 0{,}1371742\,\text{m/s}^2 \cdot 0{,}5217391\,\text{m}}}{0{,}1371742\,\text{m/s}^2}$$
(1p)

$t = 4{,}7657404\,\text{s} \approx 4{,}8\,\text{s}$

Bypass distance is A:s lead plus A:s travel before bypass happens.

$$s = v_a t + s_0 = 0{,}2173913\,\text{m/s} \cdot 4{,}7657404\,\text{s} + 0{,}5217391\,\text{m} = 1{,}5577697\,\text{m} \approx 1{,}56\,\text{m}$$

<u>Answer: B bypasses A after 4,8 s from B:s start, at 1,56 m from the end of the track.</u> (1p)

c) Bypass must happen during the time that it takes for B
to travel the whole track's length. Ultimately, bypass takes
place at time t = 5,4 s after the start of the carts, when B is at s = 2,00 m. (1p)

We calculate how long lead s_e must A have, in order to also be at location s at time t. (1p)

$s - s_e = v_a t$

$s_e = s - v_a t = 2{,}00\,\text{m} - 0{,}2173913\,\text{m/s} \cdot 5{,}4\,\text{s} = 0{,}8260870\,\text{m} \approx 0{,}83\,\text{m}$

Answer: cart A must have more than 0,83 m lead. (1p)

Form 11, problem 1

a) When the carpet has just started moving, friction with the floor has had no time to change the motion of the carpet and the dog. They can be considered as an isolated system, whose momentum is conserved. The dog's stopping on the carpet is an inelastic collision, thus kinetic energy is not conserved.

$$p_{ka}+p_{ma}=p_{kl}+p_{ml}$$
$$m_k v_{ka}+0=(m_k+m_m)v_l$$
$$v_l=\frac{m_k v_{ka}}{(m_k+m_m)}=\frac{6{,}9\,\text{kg}\cdot 7{,}8\,\text{m/s}}{6{,}9\,\text{kg}+4{,}2\,\text{kg}}=4{,}8486486\,\text{m/s}$$

<u>Answer: velocity of the dog and the carpet is 4,85 m/s.</u>

(conservation of momentum: 1p, answer 1p

b) Friction force does negative work on the dog-carpet system, and translates their kinetic energy to thermal energy.

$W = Fs$ $\quad E_k = ½mv^2$ $\quad W = E_k$

$$F_\mu s=\frac{1}{2}(m_k+m_m)v_l^2$$
$$\mu_k\cdot(m_k+m_m)g\cdot s=\frac{1}{2}(m_k+m_m)v_l^2$$
$$\mu_k=\frac{v_l^2}{2gs}=\frac{(4{,}8486486\,\text{m/s})^2}{2\cdot 9{,}81\,\text{m/s}^2\cdot 5{,}3\,\text{m}}=0{,}22608230$$

<u>Answer: coefficient of kinetic friction is 0,23.</u>

(work - kinetic energy theorem: 1p, $F_\mu s=\frac{1}{2}(m_k+m_m)v_l^2$ 1p, answer 1p)

c) The dog is accelerated by the force that the carpet exerts to the dog, F_k. The carpet does not move, which means that the forces applied to it cancel each other out (N I). The horizontal forces are the static friction between the carpet and and the floor, $F_{\mu s}$, and the force that the dog exerts to the carpet, $-F_k$ (N III).

(1p)

$$\sum \vec{F}=0$$
$$-F_k+F_{\mu s}=0$$

The maximum acceleration is achieved when
the static friction reaches its maximum. (1p)

$$m_k a = \mu_s (m_k + m_m) g$$

$$a = \frac{m_k + m_m}{m_k} \mu_s g = \frac{6{,}9\,\text{kg} + 4{,}2\,\text{kg}}{6{,}9\,\text{kg}} \cdot 0{,}30 \cdot 9{,}81\,\text{m/s}^2 = 4{,}7343913\,\text{m/s}^2$$

Answer: the maximum acceleration of the dog is 4,7 m/s². (1p)

d) The acceleration of the dog is $a = \frac{F_k}{m_k}$, which has no upper limit set by Newtonian mechanics. (Static or kinetic friction between the carpet and the floor affects only to the acceleration of the carpet.)

(answer: 1p, explanation: 1p)

Form 11, problem 2

a) When the nozzle is open, the force needed to pull out the piston equals with the kinetic friction.

When the nozzle is closed, there is near vacuum inside the syringe, which means that the pressure is almost zero. The forces exerted to the piston now are kinetic friction plus the force exerted by the atmospheric pressure. Therefore the total force is bigger when the nozzle is closed than when it is open.

(vacuum & atm pressure mentioned: 1p, answer+the rest of explanation: 1p)

Both kinetic friction and the force caused by the atmospheric pressure are constants. Therefore, the force acting on the piston, and needed to pull it out, does not change when the inner volume increases.

(answer+explanation: 1p)

b) When the piston is moving at constant speed, forces exerted on it cancel each other out (N I).

(1p)

They are the force that is pulling the piston F_{tot}, kinetic friction F_{μ}, and the force exerted by the atmospheric pressure F_{atm}.

$$F_{tot} = F_{\mu} + F_{atm}$$
$$F_{atm} = 24{,}8\,\mathrm{N} - 5{,}7\,\mathrm{N} = 19{,}1\,\mathrm{N}$$
$$p_{atm} = \frac{F_{atm}}{A} = \frac{F_{atm}}{\pi(d/2)^2} = \frac{24{,}8\,\mathrm{N} - 5{,}7\,\mathrm{N}}{\pi\cdot(0{,}0154\,\mathrm{m}/2)^2} = 102{,}54206\,\mathrm{kPa}$$

Answer: the atmospheric pressure is 103 kPa. (1p)

c) Because temperature stays constant, air inside the syringe obeys Boyle's Law: $p_{atm}V_2 = p_5V_5$.

(1p)

$$p_5 = \frac{p_{atm}V_2}{V_5} = \frac{102542{,}06\,\mathrm{Pa}\cdot 2{,}0\,\mathrm{mL}}{5{,}0\,\mathrm{mL}} = 41016{,}825\,\mathrm{Pa}$$ (1p)

$$F_{tot} = F_{\mu} + F_{atm} - F_5$$
$$F_{tot} = F_{\mu} + F_{atm} - p_5 A$$
$$F_{tot} = 5{,}7\,\mathrm{N} + 19{,}1\,\mathrm{N} - 41016{,}825\,\mathrm{Pa}\cdot\pi\cdot(0{,}0154\,\mathrm{m}/2)^2 = 17{,}16\,\mathrm{N}$$

Answer: the force required for pulling is 17,2 N (1p)

d) $p_{atm}V_{10} = p_2V_2$

$$p_5 = \frac{p_{atm}V_{10}}{V_2} = \frac{102542{,}06\,\text{Pa}\cdot 10{,}0\,\text{mL}}{2{,}0\,\text{mL}} = 512710{,}31\,\text{Pa}$$ (1p)

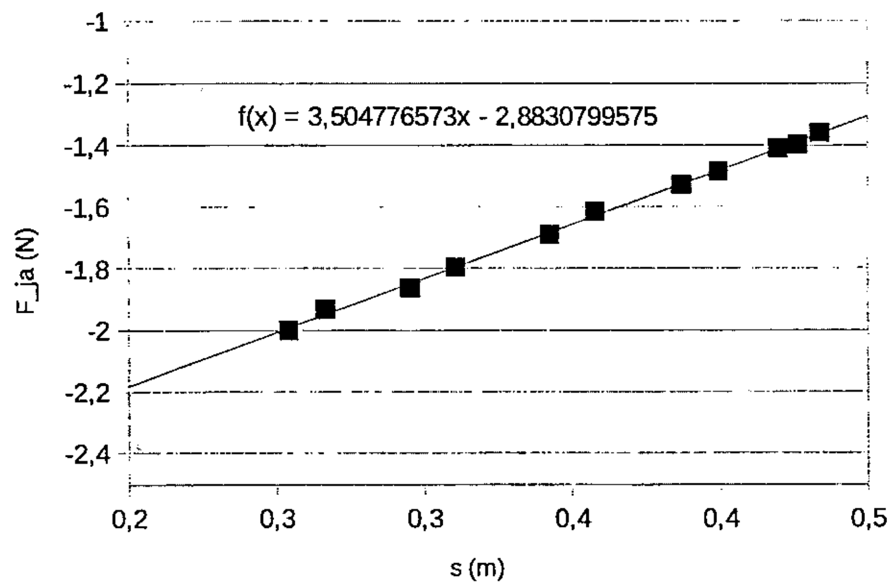
$$F_{tot} = F_2 - F_\mu - F_{atm}$$
$$F_{tot} = p_2 A - F_\mu - F_{atm}$$
$$F_{tot} = 512710{,}31\,\text{Pa}\cdot\pi\cdot(0{,}0154\,\text{m}/2)^2 - 5{,}7\,\text{N} - 19{,}1\,\text{N} = 70{,}7\,\text{N}$$

Answer: the force required for pushing is 70,7 N. (1p)

Form 11, problem 3

The force sensor records the force exerted to it by the spring, F_{ja}, which has opposite direction to the force that the spring exerts to the weight, F_{jp}. The spring is light, which means that the absolute values of these forces are equal, $-F_{ja} = F_{jp}$.

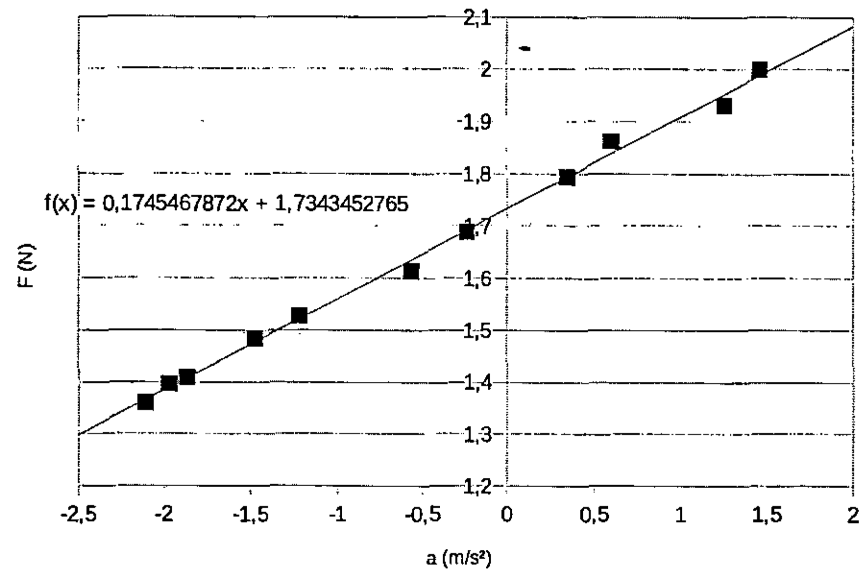
a) Pairs of $s(t)$ and $F_{ja}(t)$ are read from the graphs, and a $F_{ja}(s)$ graph is drawn. The (s,F_{ja}) pairs lie on a straight line, which means that the spring obeys Hooke's law $F = -kx$, where x is the displacement of the spring, and k is the spring constant, which is the slope of a straight line fitted in the (s,F_{ja}) pairs.



Aswer: the spring constant is 3,5 N/m:

(graph: 1p, explanation why the slope is k: 1p, value of k: 1p)

b) N II for the weight says $\sum F = ma \Rightarrow F_{jp} + G = ma \Rightarrow -F_{ja} = ma - G$. Graph of $(a, -F_{ja})$ pairs is drawn, and a straight line is fitted to the pairs. The slope of the line is the mass of the weight.



<u>Answer: the mass of the weight is 175 g.</u>

(graph: 1p, explanation why the slope is *a*: 1p, value of k: 1p)

c) Frequency of a harmonic oscillator is

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{3{,}5047766\ \mathrm{N/m}}{0{,}1745468\,\mathrm{kg}}}=0{,}7131721\ 1/\mathrm{s}$$

Period of oscillation, read from the $v(t)$ graph, is T = (1,90 - 0,52) s = 1,42 s, thus the frequency is $f = 1/T = 0{,}704\ 1/\mathrm{s}$.

<u>Answer: frequency predicted is 0,713 Hz, frequency measured is 0,704 Hz.</u> The small difference may be caused by the mass of spring, which lowers the actual frequency from what's predicted.

(predicted frequency: 1p, measured ferquency + meaningful explanation of discrepancy: 1p)

d) When there are two springs as shown, their forces change equal amounts as a function of the displacement of the spring. This has the same effect as doubling the spring constant.

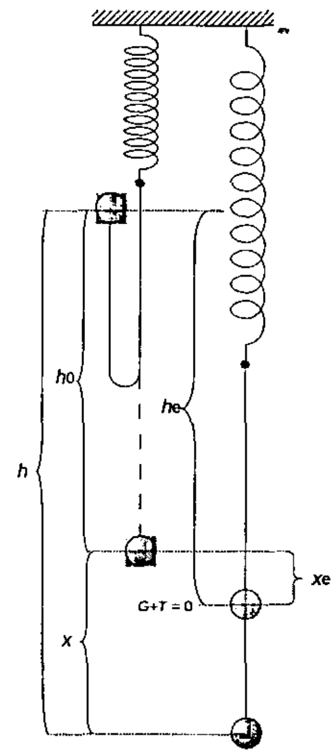
(1p)

$$f=\frac{1}{2\pi}\sqrt{\frac{2\cdot k}{m}}=\frac{1}{2\pi}\sqrt{\frac{2\cdot 3{,}5047766\,\mathrm{N/m}}{0{,}1745468\ \mathrm{kg}}}=1{,}0085777\ 1/s$$

Answer: the frequency predicted is 1,01 Hz. (1p)

Form 11, problem 4 &
Form 12, problem 1

Positive direction is downward. We will use symbols of the picture shown.



a) At its highest and lowest position, the ball does not move. At the lowest position, potential energy of the ball's weight has been transferred to potential energy of the spring force.

(1p)

$m = 0{,}100$ kg
$k = 5{,}90$ N/m
$h_0 = 0{,}30$ m
$g = 9{,}81$ m/s²

$$E_G = E_{kx}$$
$$mgh = \tfrac{1}{2}kx^2$$
$$mg(x + h_0) = \tfrac{1}{2}kx^2$$
$$\tfrac{1}{2}kx^2 - mgx - mgh_0 = 0$$
$$x = \frac{mg + \sqrt{(mg)^2 - 2k\,mgh_0}}{k}$$
$$x = 0{,}5232149\,\text{m}$$
$$h = h_0 + x = 0{,}8232149\,\text{m}$$

Answer: the ball falls 0,82 m before its motion turns upward. (1p)

b) Tension force of the cord is equal with the spring force, and directed upward. Tension reaches its maximum value at the lowest position of the ball, when the displacement of the spring has its maximum.

(1p)

$T = -kx = -3{,}0869679$ N

Answer: maximum tension of the cord is 3,1 N. (1p)

c) Two forces act on the ball: tension of the cord and gravity. They act on opposite directions. Gravity is constant, tension is either zero or depends linearly on the position of the ball. Therefore, the maximum total force and maximum acceleration must be reached at either end of the ball's travel.

At the highest position, acceleration of the weight is $g = 9{,}81$ m/s².

At the lowest position, N II states:

$$\sum F = G + T = ma$$
$$a = \frac{mg + T}{m}$$
$$a = -21{,}059679\, m/s^2$$

<u>Answer: the maximum absolute value for acceleration is 21 m/s².</u>

(acceleration: 1p, explanation of why its reached at the lowest position: 1p)

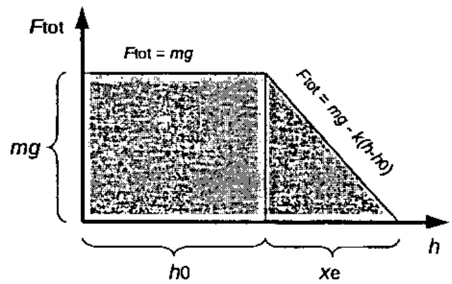
d) Velocity is increased as long as the total force exerted on the ball is downward, along the motion. Direction of force changes at the equilibrium, when the weight of the ball equals with the tension force.

(1p)

$$G + T = 0$$
$$mg - kx_e = 0$$
$$x_e = \frac{mg}{k}$$
$$x_e = 0{,}1662711\,\text{m}$$
$$h_e = h_0 + x_e = 0{,}4662711\,\text{m}$$

<u>Answer: velocity of the weight reaches maximum when the ball has dropped 0,46 m.</u> (1p)

e) When the ball drops at the equilibrium point, work done to it by the total force is the area marked on the figure.



$$W = mgh + ½mg\,x_e$$ (1p)

$$W = \Delta E_k = ½mv^2$$
$$mg(h + ½x_e) = ½mv_e^2$$
$$v_e = \sqrt{2g(h + ½x_e)} = 2{,}7417367\,\text{m/s}$$

<u>Answer: maximum velocity of the ball is 2,7 m/s</u>. (1p)

f) form 11

Mechanical energy is conserved, thus in every bounce the ball rises at the dropping level. In a harmonic oscillator, the force should be harmonic, i.e. proportional to the displacement from the equilibrium. In this system the force is not harmonic, because at the high portion of the ball's travel, the force is constant (gravity). Therefore the system is not a harmonic oscillator.

(answer: 1p, explanation: 1p)

f) form 12

We use the following symbols:

T_s : maximum tension force that the cord can take without snapping
x_s : displacement of the spring when tension is T_s
h' : max total fall
h_s: max free fall

$$x_s = \frac{T_s}{k} = \frac{2{,}5\,\mathrm{N}}{5{,}9\,\mathrm{N/m}} = 0{,}4237288\ \mathrm{m}$$

As in a):

$$E_G = E_{kx}$$
$$mgh' = \tfrac{1}{2}kx_s^2$$
$$mg(h_s + x_s) = \tfrac{1}{2}kx_s^2$$
$$mg\,h_s + mg\,x_s = \tfrac{1}{2}kx_s^2$$
$$h_s = \frac{\tfrac{1}{2}kx_s^2 - mg\,x_s}{mg}$$
$$h_s = \frac{kx_s^2}{2mg} - x_s = 0{,}1161906\,\mathrm{m}$$

Answer: maximum free fall that the cord can stand without snapping is 0,12 m.

(0,11 m is also accepted.)

(derivation + answer: 2p; minor errors with otherwise correct physics -1p)

Form 12, problem 2

a) Reading the phase diagram of CO_2 tells that in 20°C there can be either equilibrium of gaseous and liquid phases, or equilibrium of liquid and solid phases. Because part of the CO_2 is said to be gaseous, the rest must be liquid.

(answer + explanation: 1p)

b) In the tank there's only CO_2 in gaseous and liquid forms, so they must be in equilibrium, and the saturation pressure can be read from the saturation line at 20°C; it is 48 bar = 4800 kPa.

(answer: 1p, explanation: 1p)

c) CO_2 and atmospheric pressures exert opposite forces to the pellet. The net force does work on the pellet:

$$W = F_{net}\, s = (p_{CO2} - p_{atm})\, A\, s = (p_{CO2} - p_{atm}) \cdot \pi \left(\frac{d}{2}\right)^2 s$$

$$W = (4800000 - 101{,}325)\,\mathrm{Pa} \cdot \pi \cdot \left(\frac{4{,}5 \cdot 10^{-3}\,\mathrm{m}}{2}\right)^2 \cdot 0{,}23\,\mathrm{m} \qquad \text{(1p)}$$

$$W = 17{,}187715\,\mathrm{J}$$

The change in the kinetic energy of the pellet is equal to the net work done on the pellet, $W = E_k$.

(1p)

$$W = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \cdot 17{,}187715\,\mathrm{J}}{0{,}45 \cdot 10^{-3}\,\mathrm{kg}}} = 276{,}38713\,\mathrm{m/s} \approx \underline{276\,\mathrm{m/s}}. \qquad \text{(1p)}$$

d) Gravity affects the pellet, which in time t drops $h = ½gt^2$ below the boreline. Time is the pellet's time of flight.

(1p)

Actual velocity is $v_t = 0{,}60 \cdot 276{,}38713$ m/s = 165,83228 m/s.

$$h_{20C} = \frac{1}{2} g \left(\frac{x}{v}\right)^2 = \frac{1}{2} \cdot 9{,}81\,\mathrm{m/s^2} \cdot \left(\frac{10\,\mathrm{m}}{165{,}83228\,\mathrm{m/s}}\right)^2 = 0{,}0178361402\,\mathrm{m}$$

Answer: the bore must be pointed 18 mm above the centre of the target. (1p)

e) At 10°C, the saturation pressure of CO_2 is 35 bar = 3500 kPa.

$$W = (3500000 - 101325)\,\text{Pa} \cdot \pi \cdot \left(\frac{4{,}5 \cdot 10^{-3}\,\text{m}}{2}\right)^2 \cdot 0{,}23\,\text{m} = 12{,}43232\,\text{J}$$

$$v_i = \sqrt{\frac{2 \cdot 12{,}43232\,\text{J}}{0{,}45 \cdot 10^{-3}\,\text{kg}}} = 235{,}06336\,\text{m/s}$$ (1p)

The actual velocity is again 60% of the ideal velocity:
$v_t = 0{,}60 \cdot 235{,}06336$ m/s = 141,03801 m/s

Drop below the boreline is

$$h_{10C} = \frac{1}{2} \cdot 9{,}81\,\text{m/s}^2 \cdot \left(\frac{10\,\text{m}}{141{,}03801\,\text{m/s}}\right)^2 = 0{,}0246584996\,\text{m}$$

$$h_{10C} - h_{20C} = 0{,}0068223594\,\text{m}$$

Answer: at 10°C the pellets hits 6,8 mm lower than at 20°C . (1p)

Form 12, problem 3

a) Explained using work and energy:
When the circuit is open, there's no current, and no component consumes electrical energy. Therefore, to rotate the crank one only needs to overcome the friction of the mechanism. When the circuit is closed, current starts to flow and the electrical energy is transferred in the lamp into light and heat. To produce the energy in the generator one must do mechanical work by rotating the crank, and therefore cranking needs more torque..

(1p)

Explained using electric and magnetic phenomena:
When the circuit is open, there's no current, and there's no magnetic interaction between the generator coil and the magnet. When the circuit is closed, emf is induced to the generator coil. It gives rise to a current that flows through the coil. According to Lentz's law, the direction of the current is so that it's magnetic field opposes the original change of magnetic flux through the coil. This means that that magnetic interaction between the coil and the magnet tries to slow down their respective motion.

(1p)

b) Law of induction $U=-\frac{d\Phi}{dt}$ says that the induced emf is proportional to the rate of change of the magnetic flux through the generator coil, which is proportional to the angular velocity of the coil.

(law of induction + proportionalities 1p)

c) When the crank is rotated slowly, induced current starts to charge the capacitor. As explained in a), induced current tries to slow down the generator.

(1p)

As the capacitor gets charged, its terminal voltage rises. Capacitor's terminal voltage resists the induced current. The current diminishes and then stops, which ends its generator braking effect. The same steps are repeated when the rate of cranking is increased.

(1p)

(Explanations based on transferring energy into the capacitor are also accepted.)

d) When the capacitor is charging, the magnetic interaction in the generator opposes its rotation. For example, if the generator is rotated clockwise, the direction of the interaction is counterclockwise. When the capacitor discharges through the generator, the generator acts as a motor. The direction of the electric current changes. This means that the direction of the interaction also changes, so that the discharging current rotates the crank clockwise. This means that when one stops turning the crank, it continues its rotation to the initial direction.

(answer: 1p, explanation max 2p)

e) Charge and terminal voltage of the capacitor drop during discharge. In order to keep the current steady, one must adjust the resistance continuously smaller.

(1p)

Capacitance is $C=\frac{Q}{U}=\frac{I\,\Delta t}{U}=\frac{0{,}150\,\mathrm{A}\cdot 75\,\mathrm{s}}{2{,}0\,\mathrm{V}}=5{,}625\,\mathrm{F}\ \approx \underline{5{,}6\,\mathrm{F}}$ (1p)

Form 12, problem 4

Case a)

i) For a parallel plate capacitor, $U=\frac{Q}{C}, U=Ed, C=\epsilon_0\frac{A}{d}$.

Electric field between the plates is $E=\frac{U}{d}=\frac{Q}{Cd}=\frac{Q}{\epsilon_0\frac{A}{d}\cdot d}=\frac{1}{\epsilon_0}\cdot\frac{Q}{A}$. (1p)

Force exerted to the ball by the electric field is $F=Eq$; on the other hand $F=mg\sin\alpha$.

$$Eq=mg\sin\alpha$$
$$\frac{1}{\epsilon_0}\cdot\frac{Qq}{A}=\sin\alpha$$
$$\underline{q=\frac{\epsilon_0 Amg\sin\alpha}{Q}}$$
(1p)

ii) Formula $\sin\alpha=\frac{1}{\epsilon_0}\cdot\frac{Qq}{A}$ says that the angle of the string does not depend on the distance between the plates.
<u>The angle of the string does not change, when *d* is increased.</u>

(answer+explanation 1p)

iii) Energy of the capacitor is $W=\frac{1}{2}CU^2=\frac{1}{2}C\cdot\frac{Q^2}{C^2}=\frac{Q^2}{2\cdot\epsilon_0\frac{A}{d}}=\frac{dQ^2}{2\epsilon_0 A}$.

One sees that the <u>energy increases as *d* increases.</u>

(answer + explanation 1p)

<u>Energy comes from the mechanical work that done when the plates are pulled apart, against the attractive electrostatic force.</u>

(1p)

Case b)

i) Now the voltage between the plates is constant.

Force exerted to the ball by the electric field is $F=qE=\frac{qU}{d}$ (1p)

$$\frac{qU}{d}=mg\sin\alpha$$

$q = \frac{mg \sin \alpha \, d}{U}$ (1p)

ii) $\sin \alpha = \frac{qU}{dmg}$, angle of the string decreases when d is increased.

(answer + explanation 1p)

iii) $W = \frac{1}{2} C U^2 = \frac{1}{2} \epsilon_0 \frac{A}{d} U^2 = \frac{\epsilon_0 A U^2}{2d}$

The formula tells that the
energy of the capacitor decreases when d increases.

(answer + explanation 1p)

When the plates are pulled apart, capacitance decreases; therefore the charge must also decrease to keep the voltage constant. This causes electric current through the voltage source from (+) to (-). Because the voltage source is a pack of accumulators, the current charges them. Therefore the energy of the capacitor is transferred to chemical energy of the accumulators.

(answer + explanation 1p)